4. C. Crowe and H. Riesebieter, "An analytic and experimental study of jet deflection in a crossflow," AGARD Conf. Proc., No. 22, 1-19 (1967).
5. D. Adler and A. Baron, "Prediction of a three-dimensional circular turbulent jet in crossflow," AIAA J., 17, No. 2, 168-174 (1979).
6. Yu. V. Ivanov, "Certain laws governing a free circular jet developed in an external crossflow," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 8, 37-54 (1954).
7. Yu. Kamatani and I. Greber, "Experimental study of the turbulence of a jet directed into a crossflow," Raket. Tekh. Kosm., No. 11, 43-49 (1972).
8. V. Gendrikson and A. Épshtein, "Experimental study of a nonisothermal jet in a crossflow," Izv. Akad. Nauk Est. SSSR Fiz. Mat., 22, No. 3, 304-311 (1973).
9. V. A. Golubev, V. F. Klimkin, and I. S. Makarov, "Trajectory of single jets of different density propagating in a crossflow," Inzh.-Fiz. Zh., 34, No. 4, 594-599 (1978).
10. T. Okamoto and M. Yagita, "The effects of the exit profile on the flow," Bull. Tokyo Inst. Technol., No. 114, 29-47 (1973).
11. I. F. Keffer and W. D. Baines, "The round turbulent jet in a crosswind," J. Fluid Mech., 15, No. 4, 481-496 (1963).
12. S. T. Kashafutdinov and N. F. Polyakov, "Flow of a weak turbulent jet in a crossflow," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, 1, No. 3, 74-80 (1973).
13. V. A. Golubev and V. F. Klimkin, "Study of turbulent submerged gas jets of different density," Inzh.-Fiz. Zh., 34, No. 3, 493-499 (1978).

INTENSITY OF HEAT EXCHANGE NEAR A STAGNATION POINT IN THE COURSE OF PERIODIC VARIATION OF THE SURFACE TEMPERATURE

V. L. Sergeev and A. S. Strogii

UDC 536.24.01

We present the solution of the nonstationary problem of heat exchange near a stagnation point of the flow at a barrier, represented by a spherical surface, when the surface temperature of the body is periodically varied.

In practice, one often encounters the case of heat exchange between a flow and a barrier when the periodic variation of the flow parameters is close to steplike. An example is the application of obturator devices which periodically cut off the flow from a heated barrier [1]. The burning of an electric arc in a linear plasmotron with short electrodes represents blowing and ignition at high frequency [2], which leads to a periodic change of temperature of the generated jet. There are situations when heat is exchanged in the course of periodic variation of the surface temperature.

The results of investigation of the characteristics of a heat exchange with a barrier on a model of the process, represented by a steplike periodic variation of the surface temperature between arbitrary values, are given below. The full period is divided into two different time intervals during which the temperature is constant. The change of temperature takes place at the ends of the time intervals.

In the solution of this problem, we have made a preliminary study of a nonstationary heat exchange at a stagnation point of a sphere after a stepwise change of temperature of the surface (or of the flow). As in [3, 4], the flow is assumed to be subsonic, laminar, and stationary, and the properties of the liquid are assumed constant.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 4, pp. 554-558, April, 1982. Original article submitted April 9, 1981.

Using these assumptions, the system of equations has the following form:

$$\frac{\partial (xu)}{\partial x} + \frac{\partial (xv)}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \beta^2 x + \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial t}{\partial \tau^*} + u\frac{\partial t}{\partial y} + v\frac{\partial t}{\partial y} = a\frac{\partial^2 t}{\partial y^2}, \tag{3}$$

$$t(x,\ y,\ \tau^* < \tau_0^*) = t_\infty, \tag{4}$$

$$t(x,\ y = 0,\ \tau^* \geqslant \tau_0^*) = t_s, \tag{5}$$

$$t(x,\ y = \infty,\ \tau^* \geqslant \tau_0^*) = t_\infty, \tag{6}$$

$$u(x,\ 0) = v(x,\ 0) = 0,\ u(x,\ \infty) = U_i. \tag{7}$$

Introducing dimensionless variables and the flow function, and noting that the derivative ∂t/∂x in the energy equation can be neglected [3, 4] (it is zero at the stagnation point as a result of symmetry) we bring the problem to a dimensionless form for the numerical solution:

$$\theta_{k+1,i} = \theta_{k,i}\left(1 - \frac{2\Delta\tau}{\Delta\eta^2}\right) + \theta_{k,i-1}\left(1 - \frac{1}{2}\Pr f_i\Delta\eta\right)\frac{\Delta\tau}{\Delta\eta^2} + \theta_{k,i+1}\left(1 + \frac{1}{2}\Pr f_i\Delta\eta\right)\frac{\Delta\tau}{\Delta\eta^2},\ i = 1,\ 2,\ 3,\ \ldots,\ M; \tag{8}$$

$$k = 1,\ \theta_1 = 1,\ \theta_{i>1} = 0; \tag{9}$$

where

$$k > 1,\ \theta_1 = 1,\ \theta_M = 0, \tag{10}$$

$$f_i = 0.46385\eta_i^2 - 0.08333\,\eta_i^3 + 0.6442\cdot 10^{-3}\eta_i^6 - 0.4960\cdot 10^{-4}\eta_i^7 - 0.1186\cdot 10^{-4}\eta_i^9 \tag{11}$$

is the solution of the hydrodynamic part of the problem [3]; $\eta = y(2\beta/\nu)^{1/2}$; the velocity gradient for a sphere is equal to $3U_\infty/2R$; $\theta = (t - t_\infty)/(t_s - t_\infty)$, and the dimensionless time is $\tau = 2\beta\tau^*/\Pr$. The value of the number M is estimated in the course of numerical calculations from the absence of an appreciable effect on the result. The aim of the solution is to determine the characteristics of the heat exchange:

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = \sqrt{6}\,\frac{1 - \theta_{k,2}}{\Delta\eta}. \tag{12}$$

The final state of the nonstationary process under study is a stationary heat-exchange regime.

It follows from (8)-(12) that the criterion Nu in the present case of heat exchange is a function of the numbers Pr and Re, and of the dimensionless time τ. In view of the fact that the heat transfer in the body is not considered, and the heat-conduction equation is replaced, as is often in [3, 4], by a condition at the surface, the above controlling parameters do not contain characteristics of the body which, in general, affect the intensity of the nonstationary heat exchange [4, 5].

The numerical experiments were carried out on the computer "Élektronika S50" and ES 1022, for values of time step $\Delta\tau$ from 0.0025 to 0.3, coordinate step $\Delta\eta$ from 0.01 to 0.8, and the number of coordinate steps M from 40 to 400. The results were not affected substantially by the variation of parameters within the above limits, which makes it possible to choose the optimum values for the calculation of the functional dependence of $\mathrm{Nu}/\sqrt{\mathrm{Re}}$ on Pr and τ on the computer. The number Pr was varied from 0.7 to 20, and the time τ from zero to the moment when the quantity $\mathrm{Nu}/\sqrt{\mathrm{Re}}$ becomes independent of τ with error 1-2%.

In particular, it follows from the analysis of the obtained temperature fields that, for Pr = 0.7, the penetration depth of appreciable temperature variation does not exceed $\eta = 5$ during the course of the whole nonstationary heat-exchange process.

The obtained steady-state values of heat-exchange intensity depend on the number Pr (Fig. 1) which the functions $\mathrm{Nu}/\sqrt{\mathrm{Re}} = f(\tau)$ approach as the time increases. These values represent the heat-exchange characteristics in stationary conditions, and are given, in the interval of numbers Pr from 0.7 to 20, with the maximum error of the approximation 3.5% at Pr = 20, by the expression

$$\frac{\mathrm{Nu_{st}}}{\sqrt{\mathrm{Re}}} = 1.33\Pr^{0.362}. \tag{13}$$

The functional dependence (13) agrees well with the formula of M. Sibulkin obtained in the solution of a heat-exchange problem in the region of stagnation point under stationary conditions [6]:

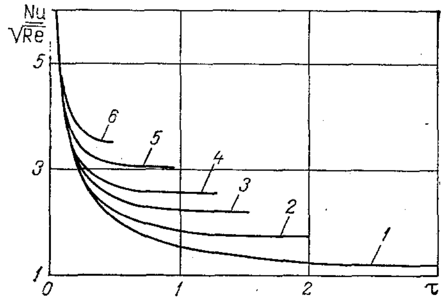

Fig. 1

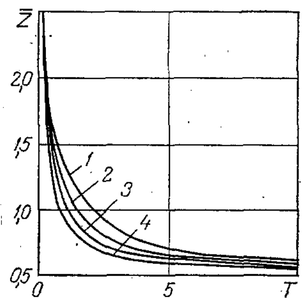

Fig. 2

Fig. 1. The combination $\mathrm{Nu}/\sqrt{\mathrm{Re}}$ as a function of the dimensionless time τ. Curve 1: Pr = 0.7, 2: Pr = 2, 3: Pr = 4, 4: Pr = 6, 5: Pr = 10, and 6: Pr = 15.

Fig. 2. The quantity $\bar{Z}=[\int_0^{kT}(\mathrm{Nu}/\sqrt{\mathrm{Re}})d\tau]/T(\mathrm{Nu_{st}}/\sqrt{\mathrm{Re}})$ as a function of the dimensionless period T for k = 0.5. Curve 1: Pr = 0.7, 2: Pr = 2, 3: Pr = 5, and 4: Pr = 10.

$$\frac{\mathrm{Nu_{st}}}{\sqrt{\mathrm{Re}}}=1.32\mathrm{Pr}^{0.4}, \tag{14}$$

for the values of the number Pr = 0.6-2. The deviation of the values of the combination $\mathrm{Nu_{st}}/\sqrt{\mathrm{Re}}$ found from (13), from those calculated using formula (14) does not exceed 2.3% in the interval of numbers Pr for which the formula of M. Sibulkin was obtained.

The results of calculation of the combination $\mathrm{Nu}/\sqrt{\mathrm{Re}}$ whose examples are given in Fig. 1, can be represented in the form of an approximate functional dependence whose error does not exceed 1% for $(\mathrm{Nu}/\sqrt{\mathrm{Re}}) \to (\overline{\mathrm{Nu}}/\sqrt{\mathrm{Re}})_{\mathrm{st}}$, and increases for $\tau \to 0$:

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}=\frac{\mathrm{Nu_{st}}}{\sqrt{\mathrm{Re}}}[1-\exp(-0.2-1.7\mathrm{Pr}^{0.35}\tau)]^{-1}. \tag{15}$$

If one defines the time of the onset of the stationary regime of the heat exchange by the condition $(\mathrm{Nu}/\sqrt{\mathrm{Re}})/(\mathrm{Nu_{st}}/\sqrt{\mathrm{Re}}) = 1.05$, the duration of the nonstationary part of the process is equal to

$$\tau_n = 1.64\mathrm{Pr}^{-0.35}. \tag{16}$$

For small values of τ at Pr = 0.7, the functional dependence of Nu/Re on τ can be represented by the expression

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}=\frac{\mathrm{Nu_{st}}}{\sqrt{\mathrm{Re}}}+0.5\tau^{-0.8}, \tag{17}$$

whose error of approximation for τ = 0.05 – 0.5 is 11-2%, respectively.

The obtained characteristics of the nonstationary heat-exchange process can be used to estimate the average characteristics of the heat exchange during a periodic stepwise temperature variation.

Let us suppose that the temperature of the surface of a body (or flow) changes at the initial moment of time discontinuously to a new constant value which causes the above nonstationary heat-exchange process. During a time interval equal to a half of the period, the temperature of the body surface changes to the original value equal to the temperature of the flow, and remains equal to this value during the second half-period during which there is not heat-exchange between the body and the flow.

The period-averaged combination $\mathrm{Nu}/\sqrt{\mathrm{Re}}$ which characterizes the heat-exchange intensity for the present model of the periodic heat-exchange process depends considerably on the dimensionless half-period, and can exceed the heat-exchange intensity in stationary conditions by a considerable factor (Fig. 2). The intensity increase or the relative value of the combination $\overline{\mathrm{Nu}}/\sqrt{\mathrm{Re}}$ increases with decreasing Pr.

The values of the period T for which the function $\bar{Z} = f(T/2)$ intersects the straight line $\bar{Z} = 1$ in Fig. 2, can be called critical values, since for $T > T_{cr}$ the heat-exchange intensity during the periodic temperature variation becomes smaller than the heat-exchange intensity of the corresponding stationary process; it tends to 0.5 $(\mathrm{Nu_{st}}/\mathrm{Re})$ with increasing period. For $T < T_{cr}$, as the frequency of the temperature variation increases, the relative average value of the combination $Nu/\sqrt{Re}$ considerably increases, and reaches three for T = 0.01 and Pr = 0.7. In other words, with increasing frequency of the temperature pulsations, the heat-exchange intensity can exceed substantially this heat-exchange characteristic in the corresponding stationary process.

In the cases when the model of the phenomenon used in the present work is applicable, the critical frequency of temperature pulsations can be estimated for Pr = 0.7-10 from the formula

$$f_{cr} = 1.68 Pr^{-0.644} U_\infty R^{-1}, \tag{18}$$

where U_∞ is the velocity of the incident flow, R is the radius of the spherical end of the axially symmetric body.

In the estimate of f_{cr} in the case of barrier in the form of flat disk of radius R, the coefficient 1.68 in formula (18) should be replaced by 0.71.

The functional dependence of $(\overline{Nu}/\sqrt{Re})/(Nu_{st}/\sqrt{Re})$ on the dimensionless period of temperature pulsations and the number Pr can be represented by the expression

$$\frac{\overline{Nu}/\sqrt{Re}}{Nu_{st}/\sqrt{Re}} = k + \frac{0{,}587}{Pr^{0{,}35}T} \ln \frac{1 - \exp(-0{,}2 - 1{,}7Pr^{0{,}35}kT)}{0{,}181}, \tag{19}$$

where $\overline{Nu}/\sqrt{Re}$ is the average value of the combination during the periodic stepwise temperature variation, and kT is the fraction of the period during which the heat-exchange is observed. In particular, in obtaining formula (18), the coefficient k should be taken equal to 0.5. As the period increases, $\overline{Z}$ tends to k.

It follows from the above data that, for example, for Pr = 1, U_∞ = 100 m/sec, R = 1 m, and k = 0.5, the critical frequency of pulsations is 168 Hz. For the frequency of temperature pulsations of 1 kHz, the coefficient of heat-exchange exceeds the heat-exchange coefficient in stationary conditions by a factor of 1.5.

LITERATURE CITED

1. E. G. Kuzovkov, V. V. Pasichnii, V. S. Dvernyakov, and V. I. Sergeev, "The effect of obturational pirometric devices on the character of sample heating in high-temperature experiments," Teplofiz. Vys. Temp., 10, No. 1, 153-156 (1972).
2. M. T. Dulei, U. K. MacGregor, and L. E. Bruier, "The characteristics of an arc in a plasma generator of the Gerdien type," Raket. Tekh., No. 9, 99-101 (1962).
3. B. T. Chao and D. R. Dzhen', "Nonstationary heat-exchange at a stagnation point," Teploperedacha, No. 2, 73-83 (1965).
4. K. Inouye and T. Joshinaga, "Transient heat transfer near a stagnation point," J. Phys. Soc. Jpn., 27, 771 (1969).
5. V. L. Sergeev, in: Heat and Mass Exchange: Physical Foundations and Methods of Investigation [in Russian], ITMO Akad. Nauk BSSR, Minsk (1980), pp. 88-90.
6. M. Sibulkin, "Heat exchange near the frontal critical point of a rotation body," Mekhanika, No. 3, 65-69 (1953).